Prove that for a full binary tree of height \( h \):
- The total number of nodes is \( n = 2^h - 1 \)
- The total number of branches is \( 2^h - 2 \)
- The total number of leaves is \( 2^{h-1} \)
- The total number of internal nodes is \( 2^{h-1} - 1 \)
- The average search cost for a node is \( O(\log n) \)

**Solution:**

*Proofs are given in the course slides.*

Draw a binary tree with 10 nodes labeled 0, 1, . . . , 9 in such a way that the inorder and postorder traversals of the tree yield the following lists: 9, 3, 1, 0, 4, 2, 7, 6, 8, 5 (inorder) and 9, 1, 4, 0, 3, 6, 7, 5, 8, 2 (postorder).

**Solution:**

![Binary Tree Diagram]

(a) A full binary tree has a total of 32767 nodes.
- What is the height of the tree?
- What is the number of internal nodes in the tree?
- What is the number of leaves in the tree?

(b) In a football cup match, the defeated team goes out. If the initial number of teams is 128, how many matches will be played to get the final winner of the cup?

**Solution:**

\( a \) \( h = \log(n+1) = \log(32768) = \log 2^{15} = 15 \), No. of internal nodes = \( 2^{h-1} - 1 = 2^{14} - 1 \), No. of leaves = \( 2^{14} \)

\( b \) No. of teams = no. of leaves = 128 = \( 2^7 \), No. of matches = No. of internal nodes = 127
Traverse the shown tree in:
- Inorder Traversal
- Preorder Traversal
- Postorder Traversal

Left as an exercise

Assume binary trees in which the leaf nodes hold integer numbers and the non-leaf nodes hold the binary operations `+`, `-`, `*`, and `/`. Provide an algorithm that, when given the root of a tree, evaluates the expression represented by the tree.

Left as an exercise