1. Solve the following recurrence relations:

\[ T(n) = 4T(n/2) + n^2 \quad \text{for } n > 1 \quad \text{with } T(2) = 1 \]
\[ T(n) = 16T(n/4) + n^2 \quad \text{for } n > 1 \quad \text{with } T(1) = 0 \]
\[ T(n) = 9T(n/3) + n^2 \quad \text{for } n > 1 \quad \text{with } T(1) = 0 \]
\[ T(n) = 2T(n/4) + \sqrt{n} \quad \text{for } n > 4 \quad \text{with } T(4) = 1, n \text{ is a power of 4}. \]
\[ T(n) = 8T(n/2) + 2n^2 \quad \text{for } n > 2 \quad \text{with } T(2) = 1, n \text{ is a power of 2}. \]
\[ T(n) = 16T(n/4) + n \quad \text{for } n > 4 \quad \text{with } T(4) = 1, n \text{ is a power of 4}. \]
\[ T(n) = 9T(n/3) + n^2 \quad \text{for } n > 3 \quad \text{with } T(3) = 1, n \text{ is a power of 3}. \]

2. Consider a recursive function to compute the largest integer less than or equal to \( \log_2 n \).

(Hint: for \( n \geq 2 \), the value of this function for \( n \) is one greater than for \( n/2 \)).

```c
int Log2(int n)
{
    if (n == 1) return 0;
    else return (1 + Log2(n/2));
}
```

Find \( T(n) \) = number of arithmetic operations.

3. Consider a recursive function to return the Number of Binary Digits in the Binary Representation of a Positive Decimal Integer \( n \) using a D&Q recursive algorithm.

```c
int BinDig (int n)
{
    if (n == 1) return 1;
    else return (1 + BinDig(n/2));
}
```

Find \( T(n) \) = number of arithmetic operations.

4. Consider a function to return the maximum value in locations \( s \) to \( e \) in an array \( a[] \).

Taking \( T(n) \) to be the number of array element comparisons, compare between the Exclude & Conquer and the D&Q algorithms to implement this function.

5. The following function receives a pointer \( (t) \) to a binary tree and returns a pointer to an exact copy of the tree:

```c
treeNode *CopyTree ( treeNode *t )
{
    treeNode *p;
    if (t)
    {
        p = new treeNode;
        p->left = CopyTree(t->left);
        p->right = CopyTree(t->right);
        p->info = t->info;
        return p;
    }
    else return NULL;
}
```

Find the number of visits for a full tree of size \( n \) nodes.
6. Consider the following function:

```c
void Doit ( itemType a[], int n, int x, int y)
{
    if ( n > 1 )
    {
        Process ( a, n, x, y);
        if ( x > y ) Doit ( a, n/2, x, y);
        else Doit ( a, n/2, y, x);
    }
}
```

If the module `Process` performs $n^2$ floating point arithmetic operations, what will be the number of such operations performed by `Doit`?

7. (a) The 1-D Discrete Fourier Transform (DFT) of $N$ samples of a signal $f(x)$ sampled at $x = 0,1,2, \ldots, N-1$ is:

$$F ( u ) = \frac{1}{N} \sum_{x} f(x) \exp \{- 2 \pi j u x / N \}$$

for $u = 0,1, \ldots, N-1$, and $j = \sqrt{-1}$. If $f(x)$ is generally complex, how many complex multiplications are needed to compute the Fourier Transform of the given sample?

(b) A significantly more efficient algorithm for computing the DFT is called the Fast Fourier Transform (FFT). This method was introduced by J. W. Cooley and J. W. Tukey in 1965. The FFT algorithm has the following recurrence relation:

$$T(n) = 2 T(n/2) + n \quad \text{for } n > 1 \quad \text{with } T(1) = 1 \ , \ n \text{ is a power of 2.}$$

Find the complexity of the FFT.

8. An array of size $(n)$ contains floating point values. Implement a Divide & Conquer algorithm to return the number of values greater than a given value $(x)$. Analyze your algorithm to find the number of floating point arithmetic operations $T(n)$ needed to achieve the result. Assume that $n = 2^m$ where $m = 0,1,2,\ldots$

9. Implement a recursive function `average(a, s, e)` utilizing Divide and Conquer to receive an array `a[ ]` of floating point numbers, the lower bound `(s)`, and the upper bound `(e)` and to return a single value representing the average of these numbers. What is the number of floating point arithmetic operations done by this function if invoke as $x = \text{average}(a, 1, n)$? Consider $n$ to be a power of 2.

10. The array of integers: $(4, 4, 8, 5, 3, 6, 1)$ is input to the Mergesort algorithm.
   - Draw the tree of the recursive calls made.
   - What is the complexity of the algorithm when sorting an array of $N$ equal elements?

11. The array of integers: $(4, 4, 8, 5, 3, 6, 1)$ is input to the Quicksort algorithm that uses the Median-of-Three as pivot.
   - Display the array step by step as it is sorted by the algorithm.
   - What is the complexity of Quicksort when sorting an array of $N$ equal elements?
12. For a binary tree, write D&Q algorithms to:
   • Return the total number of nodes in the tree.
   • Return the height of the tree.
   Find the complexity of each of these algorithms.


14. Provide an analysis of the cost of the D&Q algorithm for the Closest Pair Problem in 1-D.